Investment in Human Capital under Endogenous Asymmetric Information*

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Abstract

When is general training under-provided? We study this classic question in a model of a competitive labour market. Workers vary in firm-specific and general skills. Firms’ choices of information disclosure play a key role. Disclosing general human capital information on bad matches, but revealing nothing about good matches, leads to an efficient allocation of workers. This also creates adverse selection that enables workers to pay for efficient training. This information structure resembles the outplacement support commonly found in professional services firms. Moreover, it implies that wages of released workers can be higher than wages of those who are retained.

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1 Introduction

Pigou (1912) conjectured that firms would not provide a socially optimal level of training for their workers. Work by Becker (1962, 1964) later qualified this claim. Drawing a distinction between general and firm-specific human capital, Becker argues that, in competitive labour markets, it is the worker, not the firm, who has to pay for general training. Hence, general training might be under-provided if workers face credit constraints.

Becker’s argument that workers pay for training is founded on perfect competition, in particular multiple price-taking firms all holding symmetric information. Prompted by evidence of apprenticeships in Germany and elsewhere, Acemoglu and Pischke (1998) subsequently explored whether firms have incentives to invest in general training in labour markets that are imperfect. These authors show that, when current employers have better information than potential employers about a worker’s ability, the wage structure becomes compressed via adverse selection and this encourages firms to sponsor general training.¹ Consequently, if labour markets feature asymmetric information, general training can be provided even when workers are credit constrained.

Despite this long tradition, there remains an open question: will labour markets feature the ‘right kind’ of asymmetric information to ensure that general training is optimally provided? The answer is not obvious given a potential trade off between productive efficiency in training and allocative inefficiency caused by adverse selection. Our paper addresses this question.

We develop a model of general training in which the nature and extent of asymmetric information (in an otherwise competitive labour market with credit-constrained workers) is endogenous. In our model, multiple (ex ante identical) firms compete to hire a worker over two periods. During the first period, the firm that has successfully hired the worker, the first-period employer, can incur a cost to provide the worker with training that augments her human capital. If she is retained, her output in the second period depends on her innate general human capital, her training, and the

¹Katz and Ziderman (1990) and Chang and Wang (1996) focus on asymmetric information over the amount of training itself, rather than the worker’s ability. In these models, it is the resulting monopsony power that encourages firms to sponsor general training. Other kinds of labour market frictions can serve a similar role. See, for instance, Almazan, de Motta, and Titman (2007) who show that a firm’s choice of location can introduce labour market frictions that allow for training.
quality of the match with the first-period employer. This match value can be positive or negative, implying that turnover can have implications for allocative efficiency.\(^2\) If the worker is released, her output at a rival firm depends on her innate general human capital and any training at the first-period employer. At the start of the first period, firms compete through contract offers that consist of a non-negative starting wage (to capture credit constraints), a statement specifying whether training will be provided, and a disclosure policy (a function from the worker’s two-dimensional productivity type to a publicly observable signal).\(^3\) Following the literature on Bayesian persuasion and information design, we assume that firms can commit to their disclosure policy and place no substantive restrictions on the set of possible signals (c.f. Bergemann and Morris, 2019; and Kamenica, 2019).

In our set up, general training is efficiently provided if it takes place whenever the addition to the worker’s innate general human capital exceeds the cost to the firm. We explore whether firms offer contracts with training in this case. We show that, if the cost of training is sufficiently low, an equilibrium exists in which all firms offer contracts with a commitment to provide training and to publicly disclose the realization of the worker’s innate general human capital. The worker pays for training; compared to the counterfactual without training, her starting wage is lower by an amount equal to the cost of training. This is what one might call the ‘Becker equilibrium’; there is (endogenously chosen) symmetric information, efficient provision of general training, and allocatively efficient turnover.

As the cost of training rises, a cut off is reached and this equilibrium no longer exists. This is because the worker’s credit constraint binds and the starting wage cannot be lowered by enough to cover the cost of training. But the Pigou conjecture does not hold. As long as the cost of training is not too high, an equilibrium exists

\(^2\)This potential for allocatively inefficient turnover can be contrasted with some of the extant literature on adverse selection in labour markets. For example, in Greenwald (1986) there is exogenous turnover but otherwise workers are equally valuable at all firms. In Waldman (1984), misallocation is through task assignment rather than by firm. Ferreira and Nikolowa (2020) consider ex ante heterogeneous firms leading to a job ladder, so that there may be inefficient turnover.

\(^3\)Previous work has highlighted that firms might actively design information structures to manage their human capital (Koch and Peyrache, 2005; Mukherjee, 2008; Strobl and van Wesel, 2013; Bar-Isaac and Lévy, 2021). Our contribution is to study training and to allow for richer information. We build on the model in Bar-Isaac et al (2020), adding a first period in which firms take decisions about how much general training to provide, and what type of information to disclose. Carter (2020) also studies training in a model with firm-worker matches values but focuses on dynamics, as information on the match is (exogenously) revealed over time. In a similar vein, Garicano and Rayo (2017) study relational incentives to provide training, again assuming exogenous information.
in which all firms offer contracts with a commitment to provide training and to disclose the realization of the worker’s innate general human capital if and only if she is (internally) found to be a bad match—what we term full disclosure on bad matches, FDBM. Thus, these ill-matched worker types suffer no adverse selection and earn a wage that fully reflects their general ability. Instead, no information is revealed regarding workers who are good matches: adverse selection applies in full force and their wages are driven below their average ability. This FDBM equilibrium features (endogenously chosen) asymmetric information, efficient provision of general training, and allocatively efficient turnover. The worker still pays for training. One might ask how, since she remains credit constrained and the cost of training is higher than before. The answer is that she pays for training ex post via a lower second period wage. In effect, FDBM allows the worker to pledge as much future income as possible in a way that involves no efficiency loss in the form of allocative distortion.

In sum, we show that the ‘right kind’ of asymmetric information can arise endogenously in competitive labour markets and, in doing so, ensure that general training is optimally provided, even in the presence of worker credit constraints. This endogenous asymmetric information, based on multi-diminensional productivity types, is not of the kind assumed by Acemoglu and Pischke (1998), or indeed of the type envisaged in the literature testing for adverse selection in labour markets which has seen mixed success (Gibbons and Katz, 1991; Lang and Weinstein, 2016). But it does have empirical support. In providing considerable information when things do not work out but little otherwise, FDBM is consistent with human resource practices observed in the professional services sector. Specifically, it is reminiscent of outplacement support (career guidance, introductions, testimonials, and reference writing) to help ill-matched or unsuitable workers find external offers. Such activities are common in professional service firms where human capital and training are particularly important. For example, Gilson and Mnookin (1989) point to outplacement support in corporate law firms with up-or-out policies. As a specific illustration, on its own recruitment page, consulting firm Bain and Co highlighted the considerable support given to associate consultants moving on after “just two or three years.”

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4 The FDBM equilibrium does not always exist. As the cost of training rises, eventually a second cut off is reached and the worker’s credit constraint binds. However, as we discuss below, in a parametric variant of the model, an equilibrium that resembles the FDBM equilibrium always exists.

5 The recruitment page https://www.bain.com/careers (accessed 22 April 2019) noted that: “We want our employees to thrive at Bain, regardless of what their future plans are. Our dedicated
Our results highlight a link between firms’ decisions about information structures, such as outplacement activities, and their decisions about training. These come together to make a position more attractive to a potential employee and are a part of a broad human capital management strategy that aims to recruit, develop and retain the right talent—the key strategy for human-capital-intensive firms in the professional services sector and beyond (Maister, 1997).

2 Model

There are two periods. At the start of the first period, at least three identical firms compete to hire a worker. We refer to the firm that successfully hires the worker as the first-period employer, and all others as rival firms. The rival firms compete to attract the worker away from the first-period employer at the start of the second period.

Productivity The worker does not produce output during the first period but the first-period employer can provide her with training to augment her human capital. This training increases the worker’s productivity at any firm in the second period by $a$ and incurs cost $c$ in the first period. The notation $1_{tr}$ denotes an indicator function that takes the value 1 when training is provided, and the value 0 otherwise.

If the worker is retained by the first-period employer, her output during the second period is determined by training (if provided in the first period) and her innate productivity type, denoted by $(G, M)$ where $G$ is a random variable representing general human capital and $M$ is a random variable representing the match value at the first-period employer. We assume the support of $(G, M)$ is a finite set in $\mathbb{R}^2$. Specifically, the worker’s realized general human capital $G = g$ is drawn from $G = \{g_1, ..., g_n\}$, where the minimum element $g_1$ is normalised to zero and $n \geq 2$. Her realized match value $M = m$ is drawn from $M = \{m_1, ..., 0, ..., m_p\}$, with minimum element $m_1 < 0$ and maximum element $m_p > 0$. Hence all worker types are (weakly) productive but matches can be good or bad. We impose no distributional assumptions.
other than a positive probability of all realizations \((g, m)\). The first-period employer observes the worker’s realized productivity type \((g, m)\) during the first period and knows that she will produce \(g + m + a_1 tr\) if she is retained for the second period.

If the worker is released by the first-period employer, her output during the second period is determined by training (if provided in the first period) and her innate general human capital; i.e., it is \(g + a_1 tr\). As we now discuss, rival firms observe whether training is provided together with a public disclosure from the first-period employer.

**Contracts** At the beginning of the first period, firms offer contracts that consist of a commitment specifying whether or not training will be provided, a commitment to a disclosure policy \(\mu\), and a non-negative starting wage \(w_s\). The disclosure policy determines how much of the first-period employer’s private information is publicly revealed to rival firms at the end of the first period. Letting \(\mathcal{T}\) denote a sufficiently large set of possible signals, a (non-random) disclosure policy is a function from the set of innate productivity types to the set of possible signals, \(\mu : \mathcal{G} \times \mathcal{M} \rightarrow \mathcal{T}\). Thus, for a given type realization \((g, m)\), the disclosure policy \(\mu(g, m) = t\) gives a realisation of public information \(t\).

At the beginning of the second period, the rival firms compete to attract the worker away from the first-period employer by making wage offers that are no lower than the lowest possible level of productivity, \(w \geq a_1 tr\). The first-period employer, on the basis of its (possibly) private information, then chooses whether to match the highest offer and retain the worker, or to release her. When the highest outside offer is equal to \(g + m + a_1 tr\), the first-period employer is indifferent; for simplicity, we assume that the first-period employer matches the offer and the worker is retained.

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6For simplicity, we assume that there is no match value at rival firms. Nothing would change if we added such a component, as long as it is an independent random variable with zero mean.

7We focus on non-random disclosure policies for simplicity. It will become clear that this is without loss of generality (there is no gain, but additional notation, associated with randomization). In our exposition in Section 3, it suffices to focus on two particularly simple policies: full disclosure (of general human capital) and full disclosure on bad matches. In doing so, we will require \(|\mathcal{T}| \geq n+1\).

8We discuss the role of this lower bound on wages in Section 3 (see footnote 10).

9Competition in the second period therefore follows the procedure set out by Greenwald (1986), which has been adopted by much of the subsequent literature, including Gibbons and Katz (1991) and Acemoglu and Pischke (1998). Alternatives to this wage-setting protocol include Pinkston (2009), who studies ascending ‘button’ auctions, and Li (2012), who studies first-price auctions. For an overview of the literature on asymmetric information in labour markets, see Waldman (2017).
To summarize, the timing is as follows.

**First Period** Each firm simultaneously offers a starting contract. The worker chooses one of these contracts and is paid the contracted starting wage. Training is provided or not, and the cost of training (if provided) is incurred. The first-period employer privately observes $g$ and $m$, and the rival firms observe the disclosure policy and the realization of public information, in addition to whether training has been provided.

**Second Period** Rival firms simultaneously post wage offers. The first-period employer observes these outside offers and then makes a wage counter-offer. The worker chooses one of these offers and is then paid the associated wage. Production takes place and payoffs are realized.

Our solution concept is Perfect Bayesian Equilibrium (PBE). In such an equilibrium: the worker’s starting contract choice maximizes her lifetime expected wages, taken as the simple sum of her starting wage and expected second-period wage (for simplicity, there is no discounting between periods); each firm’s starting contract offer maximizes its expected profit (again with no discounting between periods); and in the second period, the wage offer of each rival firm maximizes its expected profit given public information and its updated beliefs of the worker’s type, taking into account the strategies of other firms.

3 Analysis

We can establish our main results by focusing on two simple disclosure policies that achieve an efficient allocation of the worker: full disclosure (of general human capital), and full disclosure on bad matches. When discussing these policies, we will take the signal space to be $\mathcal{T} = \mathcal{G} \cup \{\text{good match}\}$.

3.1 Second-period outcomes under full disclosure

This disclosure policy takes the form

$$\mu^{\text{Full}}(g, m) := g.$$
For any innate productivity realisation \((g, m)\), public information is \(t = g\); i.e., the first-period employer discloses the worker’s general human capital. In the event that the worker chooses to train at a firm committed to full disclosure, we can establish the following features of the equilibrium in the second-period continuation game.

**Proposition 1.** Suppose the worker trains at a firm committed to full disclosure. Then, in the second period, there is a unique PBE. The equilibrium wage at the realization of public information \(t\) is \(w(t) = g + a_{1tr}\). The first-period employer matches this offer and retains the worker if and only if \(m \geq 0\). In the event that the first-period employer does retain the worker, it makes a second-period profit of \(m\).

*Proof.* In the second period, competition between the rival firms leads them to bid up to their expected value of employing the worker. Since rival firms observe both training and the worker’s innate general human capital, they bid up to \(g + a_{1tr}\). The first-period employer will match this wage and retain the worker when her productivity is at or above this wage this level; i.e., if and only if \(g + m + a_{1tr} \geq g + a_{1tr}\), or, equivalently, \(m \geq 0\). This establishes that the worker is efficiently allocated. In the event that the first-period employer does retain the worker, it earns a profit equal to the worker’s output, \(g + m + a_{1tr}\), less the wage, \(g + a_{1tr}\), and so profit is simply \(m\). \(\square\)

We now use Proposition 1 to state a remark that collects together the ex ante expected values of second-period outcomes.

**Remark 1.**

1. Expected second-period profit. The ex ante expectation of the second-period profit that a firm would earn if it offered a starting contract with a commitment to full disclosure that was accepted by the worker is \(E[M | M \geq 0] \Pr[M \geq 0]\).

2. Expected wage. The ex ante expectation of the second-period wage that the worker would receive having accepted such a contract is \(E[G] + a_{1tr}\).

3. Expected second-period surplus. The ex ante expectation of the surplus that would be generated in the second period if the worker accepted such a contract is

\[
E[G] + \underbrace{E[M | M \geq 0] \Pr[M \geq 0]}_{\text{maximum contribution to surplus}} + a_{1tr}.
\]
The expressions in Parts 1 and 2 follow straightforwardly from Proposition 1. By virtue of competition, the rival firms all make zero profit in the second period. Hence expected second-period surplus is the sum of these two quantities. The first term in (1) is the contribution to surplus from the worker’s innate general human capital and the match value, and is therefore a measure of allocative efficiency. Note that, since no bad matches and all good matches are retained, this is the maximum contribution to expected second-period surplus that could be achieved—i.e., maximum allocative efficiency.

3.2 Second-period outcomes under FDBM

This disclosure policy takes the form

$$\mu^{FDBM}(g, m) := \begin{cases} g & \text{if } m < 0 \\ \text{good match} & \text{if } m \geq 0. \end{cases}$$

If the worker is a bad match, the first-period employer discloses the worker’s general human capital; otherwise, rival firms learn that the work is a good match. In the event that the worker chooses to train at a firm committed to FDBM, we can establish the following features of the equilibrium in the second-period continuation game.

Proposition 2. Suppose the worker trains at a firm committed to FDBM. Then, in the second period, there is a unique PBE. The equilibrium wage at the realization of public information $t$ is

$$w(t) = \begin{cases} g + a_{1tr} & \text{if } t = g \\ a_{1tr} & \text{if } t = \text{good match}. \end{cases}$$

The first-period employer retains the worker if and only if $m \geq 0$. In the event that the first-period employer does retain the worker, it makes a second-period profit of $g + m$.

Proof. Suppose the worker is a bad match, $m < 0$. In this case, the wage is determined just as under full disclosure: rival firms observe the realization $g$ and so bid the wage up to $g + a_{1tr}$. Since this wage is higher than the worker’s productivity at the first-period employer, $g + m + a_{1tr}$, she is released.
Now suppose the worker is not a bad match, $m \geq 0$. The only thing that rival firms learn about the worker is that she is at least as productive at the first-period employer as she would be at their own firm. Consequently, rival firms understand that if they make an offer and the first-period employer does not wish to retain the worker at this wage, then this offer is too high. This leads the rival firms to bid lower. It is only once the wage has reached the lowest possible general human capital realization ($g_1$, which we have normalized to zero), plus any training, that outside firms no longer need to be concerned that the worker will produces less than their offer.\footnote{Note that we are using the assumption that rival firms cannot offer less than $a_{1_{tr}}$. Without this assumption, there can be other PBE. In particular, if $1_{tr} = 1$, then any wage in the range $[0, a]$ can be sustained as an equilibrium when $t = \text{good match}$; rival firms are certain that they will be unable to attract the worker and so it is irrelevant which losing offer they make. Such equilibria lead to similar qualitative insights. Moreover, allowing for trembles in the first-period employer’s behaviour would select the equilibrium described in the statement of the proposition (because rival firms Bertrand compete in the event of the first-period employer’s tremble).} Since this wage is never higher than the worker’s productivity at the first-period employer, she is retained. In the event that the first-period employer does retain the worker, it earns a profit equal to the worker’s output, $g + m + a_{1_{tr}}$, less the wage, $a_{1_{tr}}$, and so profit is $g + m$.

We now use Proposition 2 to state a remark that collects together the ex ante expected values of second-period outcomes.

**Remark 2.**

1. **Expected second-period profit.** The ex ante expectation of the second-period profit that a firm would earn if it offered a starting contract with a commitment to FDBM that was accepted by the worker is $\mathbb{E}[G + M \mid M \geq 0] \Pr[M \geq 0]$.

2. **Expected wage.** The ex ante expectation of the second-period wage that the worker would receive having accepted such a contract is $\mathbb{E}[G \mid M < 0] \Pr[M < 0] + a_{1_{tr}}$.

3. **Expected second-period surplus.** The ex ante expectation of the surplus that would be generated in the second period if the worker accepted such a contract is given in (1).

The expressions in Parts 1 and 2 follow directly from Proposition 2. Since no bad matches and all good matches are retained, expected second-period surplus is
the same as under full disclosure (maximum allocative efficiency). Crucially for what follows, this surplus is split differently between the worker and the firm: expected second-period profit is higher, whereas the expected wage is lower, under FDBM than under full disclosure.

In this and the preceding section, we have highlighted two disclosure policies that are allocatively efficient—full disclosure and FDBM—but there are others. One example is a policy that reports both general human capital and match value \((g, m)\) for all productivity types. The following result establishes a ranking among disclosure policies that are allocatively efficient, which will be useful as we turn to first-period choice of contracts below.

**Proposition 3.** Among the set of disclosure policies that are allocatively efficient, FDBM maximises the ex ante expectation of second-period profit.

**Proof.** This is immediate on noting that in the second period, the first-period employer is indifferent to the wage of a released worker. Instead, trivially, the wage of a retained worker is minimized under FDBM. \(\square\)

### 3.3 First-period outcomes

We now characterise the equilibrium starting contracts offered by firms at the beginning of the first period. To simplify the statement of the result, we will use the shorthand notation \(\pi^{\text{Full}}\) and \(\pi^{\text{FDBM}}\) to represent expected second-period profit under full disclosure and FDBM respectively, and \(s^*\) to represent the firm term in (1)—the maximum contribution to expected second-period surplus from the worker’s innate general human capital and the match value.

**Proposition 4.**

i. If training is inefficient \((c \geq a)\), there exists an equilibrium in which firms offer the contract \(\{1_{tr} = 0, \mu^{\text{Full}}, w_s = \pi^{\text{Full}}\}\), and an equilibrium in which firms offer \(\{1_{tr} = 0, \mu^{\text{FDBM}}, w_s = \pi^{\text{FDBM}}\}\). In all equilibria, expected total surplus is \(s^*\).

ii. If training is efficient \((c < a)\) and training costs are low \((c \leq \pi^{\text{Full}})\), there exists an equilibrium in which firms offer \(\{1_{tr} = 1, \mu^{\text{Full}}, w_s = \pi^{\text{Full}} - c\}\), and an equilibrium in which firms offer \(\{1_{tr} = 1, \mu^{\text{FDBM}}, w_s = \pi^{\text{FDBM}} - c\}\). In all equilibria, expected total surplus is \(s^* + a - c\).
iii. If training is efficient \((c < a)\) and training costs are intermediate \((\pi^{FDBM} < c \leq \pi^{Full})\), there does not exist an equilibrium in which firms offer \(\{1_{tr} = 1, \mu^{Full}, w_s \geq 0\}\) but there does exist an equilibrium in which firms offer \(\{1_{tr} = 1, \mu^{FDBM}, w_s = \pi^{FDBM} - c\}\). In all equilibria, expected total surplus is \(s^* + a - c\).

iv. If training is efficient \((c < a)\) and training costs are high \((\pi^{FDBM} < c)\), then, in any equilibrium, expected total surplus is less than \(s^* + a - c\).

Proof. Parts i-iii follow from a comparison of the quantities in Remarks 1 and 2, and, in particular, on noting that \(\pi^{FDBM} > \pi^{Full}\). Part iv follows from Proposition 3.

The key to this result is the observation that, by virtue of competition, each firm designs its training and disclosure policies to maximise ex ante expected total surplus, and its starting wage to ensure that all of this surplus is transferred to the worker.

In the first case, training is not efficient and so is not provided. This leaves firms free to choose any allocatively efficient disclosure policy (including full disclosure and FDBM). In the absence of training costs, competition leads each firm sets its starting wage equal to its ex ante expected second-period profit (in the event that it hires the worker), leaving it with zero expected profit.

In the second case, training is both efficient and low cost. A firm can choose full disclosure since this allocates the worker efficiently and generates sufficient ex ante expected second-period profit to cover the cost of training. But a firm can also choose FDBM for the same reason. Each firm sets its starting wage equal to its ex ante expected second-period profit (in the event that it hires the worker) given its chosen disclosure policy less the training cost, leaving it with zero expected profit.

In the third case, training is efficient but now of intermediate cost. If a firm offers a starting contract with training and a full disclosure policy it expects to make a loss in the event that it hires the worker (because its ex ante expected second-period profit is insufficient to cover the cost of training, \(\pi^{Full} < c\), and the credit-constrained worker cannot pay towards training up front). The firm would do better to offer a starting contract without training, a full disclosure policy, and a starting wage of \(w_s = \pi^{Full}\), since this leaves it with zero expected profit. But such a contract cannot be offered in equilibrium because there is a profitable deviation. Suppose the firm offered a starting contract with training, a disclosure policy of FDBM, and a starting wage of \(w_s = \pi^{FDBM} - c - \epsilon\). It would attract the worker, since she would expect...
to gain $a - c - \epsilon$, while the firm would expect to gain $\epsilon$. In contrast, it can be an equilibrium for firms to offer a starting contract with training, FDBM, and a starting wage of $w_s = \pi^{FDBM} - c$, since all firms make zero expected profit and there are no profitable deviations. Crucially, FDBM provides a way for the credit-constrained worker to pay for training ex post.

In the final case, training is efficient but high cost. If a firm offers a starting contract with training and a disclosure policy of FDBM it expects to make a loss in the event that it hires the worker (because $\pi^{FDBM} < c$ and the credit-constrained worker cannot pay towards training up front) and so such a contract cannot be offered in equilibrium. We know from Proposition 3 that there is no other allocatively efficient disclosure policy that generates more expected second-period profit than FDBM. Hence in any equilibrium that does exist, expected total surplus must be less than $s^* + a - c$. Given the high training cost, it is not possible for a firm to offer a starting contract that provides training and allocates the worker efficiently.

Of course, this leaves open the question of what starting contract could be offered in equilibrium. Firms face a trade-off: maximize allocative efficiency and forsake the gain from training of $a - c$; or, if feasible, forsake some allocative efficiency by adopting a disclosure policy that lowers the second period wage for some bad matches (as well as good matches). The terms of this trade off—i.e., how much allocative efficiency must be given up as $c$ increases—depend on distributional parameters. No parametric assumptions, beyond a finite type space, were needed to make our basic point that FDBM can be an endogenous (first best) response to the classic problem of paying for general training. Rather than making specific assumptions here, we point the reader to Bar-Isaac et al (2020), who explore the relationship between adverse selection and efficiency in a one-period, fully parametric Gaussian model. That paper does not consider FDBM as we define it here (because public information is not normally distributed). However, a class of disclosure policy that resembles FDBM can depress the expected second-period wage to any required level with only an arbitrarily small allocative efficiency loss.\(^\text{11}\) Hence, in that specific parametric setting, our key insight stands: by revealing information about bad matches, firms can enable credit-constrained workers to pay for training ex post.

\(^{11}\)See the discussion of Proposition 9 in Bar-Isaac et al (2020). The signal in question is $T = (G - \beta(G + M), \sigma M + \epsilon)$, where $\epsilon$ is unit-normal noise. The expected second period wage can be depressed below $E[G]$ with arbitrarily little efficiency loss by reducing $\beta < 0$ and taking $\sigma \to 0$.\(^{13}\)
3.4 Released versus Retained Workers

We complete our analysis by exploring the labour market implications of the FDBM equilibrium. We present a result that addresses two natural questions: do released workers earn, on average, higher or lower wages than retained workers; and do released workers have, on average, higher or lower general human capital than retained workers?

**Proposition 5.** Suppose a firm adopts a disclosure policy of FDBM.

1. The expected wage of a worker who is released is higher than the expected wage of a worker who is retained.

2. If $G$ and $M$ are negatively dependent, then the expected general human capital of a worker who is released is higher than the expected general human capital of a worker who is retained.

**Proof.** Part 1 follows from Proposition 2; if the worker is retained she is paid $a_1 t_r$, and if she is released she is paid $g + a_1 t_r$. Negative dependence of $G$ and $M$ (Lehmann, 1966) implies $E[G|M < 0] > E[G] > E[G|M \geq 0]$, which establishes Part 2.

Under a disclosure policy of FDBM there is adverse selection: given public information $t = \text{good match}$, the wage of a retained worker is depressed by rival firms’ fear of the winner’s curse. Since adverse selection falls only on good matches, released workers earn more than retained workers. It is also possible (depending on the joint distribution of $G$ and $M$) that released workers have higher general human capital than retained workers—i.e., that there is positive selection of general human capital into the outside market.

These observations have implications for empirical work. As Bar-Isaac et al (2020) discuss, if the econometrician is unable to condition on public information, the famous Gibbons-Katz (1991) test for adverse selection becomes a test for negative selection into the outside market. Hence, if general human capital and firm-specific match value are negatively dependent or even simply independent, this version of the Gibbons-Katz test will fail to detect the adverse selection that exists under FDBM. Such logic could help to explain why the literature following Gibbons and Katz (1991) has had mixed results (Lang and Weinstein, 2016).

The prediction of a wage penalty for well-matched workers, should surprise few readers of this paper. In many universities, an outside offer is key to raising salary.
Yet rival departments are typically reluctant to make offers to academics who appear settled at their current institution (for example, because they have middle school-aged children who might not be supportive of a move; a partner settled in a career that is location-specific; or parents and extended family who have spent their lives in that location). Hence, well-matched academics, who are publicly known to be so, are often paid less than their more mobile colleagues. In a similar vein, Bidwell (2011) finds that external hires by a large investment bank are, on average, paid 18 percent more than internal promotions to identical positions. These examples support the prediction in Part 2. Of course, in other settings employer goodwill and/or employee moral hazard (dissembling to be seen as a bad match) could temper the forces highlighted in our model.

4 Discussion

We have purposefully developed a simple model. In this section, we discuss the role of commitment, both to disclosure and to the level of training provision, and related extensions.

Disclosure  In our model, players (here firms) can choose among, and commit to, disclosure policies. This is a typical assumption in the literature on information design (c.f. Kamenica, 2019). It is also plausible in our labour market application. In the software industry, firms often commit contractually (or via reputation) to the amount of time that a programmer can spend on open vs. closed-source projects, thereby affecting the amount of information available to potential employers (Lerner and Tirole, 2005). In the professional services sector, firms can either limit or encourage the extent to which a consultant or lawyer has direct access to clients, forbid or allow online publication of their work, and even govern their social interactions (Liebeskind, 1997). This tends to be done via standard contractual terms or rules at the level of the organization, rather than policies tailored to individuals. More broadly, organizational design choices such as team size, layers of hierarchy, evaluation design, and promotion criteria will affect the information available to rival firms (Waldman, 1984). It is also worth noting that we would obtain similar insights in a model where firms cannot commit to a disclosure policy but can commit to an up-or-out contract that fixes the proportion of workers that are retained. In this variant,
it could be time consistent for a firm to reveal no additional information about the (well-matched) fraction retained and, absent disclosure costs, full disclosure about the (poorly-matched) fraction released.

We have assumed that firms do not choose what information to acquire about the worker, only what to disclose. We could instead assume that firms commit to internal review procedures, as in Smolin (2021), and that all information obtained via this performance evaluation is disclosed. In this variant, FDBM would correspond to a procedure whereby the first-period employer evaluates whether the worker is a good match or not, and then follows up to evaluate general skills in the case of a bad match. Allowing both acquisition and disclosure would yield similar outcomes, albeit with multiplicity in how to achieve them.

**Training**  We have assumed a simple technology for training: the decision to provide training is binary, and the outcome is deterministic. Training is also something that firms can commit to. It is only relevant for general productivity. And all firms have access to the same training technology. A natural variant of the model would admit heterogeneity in training technologies among firms. For example, there could be one ‘blue-chip’ firm that provides high-value training and a competitive fringe of identical firms that provide lower-value training. In equilibrium, the ‘blue-chip’ firm would adopt FDBM to claw back economic rent from a credit-constrained worker.

We could also allow training to augment both general and firm-specific human capital. Introducing firm-specific training into the model creates the possibility of additional surplus that, through competition, would lead to a higher starting wage. Since this relaxes the non-negativity constraint, firm-specific training could therefore be complementary to the provision of general training. Stevens (2001), Kessler and Lulfesmann (2006) and Balmaceda (2007) also highlight strategic complementarity between firm-specific and general training, albeit through different mechanisms. All three approaches rely in some way on an imperfect labour market. Extending our model to admit firm-specific training would provide an equilibrium foundation for one ‘imperfection’ in the form of endogenous (and efficiently imposed) adverse selection.

Turning to commitment and the deterministic consequences of training, consider the following example. Suppose the worker’s general human capital is known, say $G = \{5\}$; the match value is binary, $M = \{-1, 1\}$; the outcome of training is a random variable $A$ with realization $a$ drawn from the set $\{0, 3\}$; and both random variables are...
drawn independently from discrete uniform distributions. Further, assume the same timing as in the baseline model, except that after hiring the first-period employer decides whether to incur the cost of training (i.e. there is no commitment to training).

Under full disclosure outside firms observe \((g, a)\). If the first-period employer trains: the equilibrium wage is \(g + a\), only the good match (regardless of the outcome of training) is retained, and so the first-period employer’s expected profit is \(\frac{1}{4}(6 - 5) + \frac{1}{4}(9 - 8) - c = \frac{1}{2} - c\). If the first-period employer does not train, the equilibrium wage is \(g\), again only the good match is retained, and so its expected profit is \(\frac{1}{2}(6 - 5) = \frac{1}{2}\). Trivially, profits are higher without training, hence the first-period employer does not train the worker, as under the standard Pigou conjecture. Now consider FDBM, where outside firms observe \((g, a)\) if \(m = -1\) and \(m\) otherwise. If the first-period employer trains, the equilibrium wage is \(g + a\) if \(m = -1\) and \(g\) otherwise, only the good match is retained, and so its expected profit is \(\frac{1}{4}(6 - 5) + \frac{1}{4}(9 - 5) - c = \frac{5}{4} - c\). If the firm does not train, the equilibrium wage is \(g\), only the good match is retained, and so the first-period employer’s expected profit is \(\frac{1}{2}(6 - 5) = \frac{1}{2}\). So under FDBM, the firm will provide training if the cost is sufficiently low, \(c < \frac{3}{4}\). Hence our basic insight, that FDBM can facilitate the provision of efficient training, still stands.

It is worth noting that there was no mention of credit constraints in the paragraph above. With no commitment to training, even when the non-negativity constraint on the starting wage does not bind, there is a role for disclosure policies that create adverse selection in the second period. This is because the first-period employer must anticipate a sufficient second-period return to be prepared to train the worker. A lack of commitment to training (with much of it informal), coupled with a high starting wage, and approaches to outplacement that resemble FDBM, reflects the professional services sector at least as well as the baseline model.

5 Concluding Remarks

This paper revisits the classic Pigou conjecture concerning under-provision of general training. It is well understood that, if labour markets feature asymmetric information, then general training can be provided even when workers are credit constrained. But there remains an open question: will labour markets feature the ‘right kind’ of asymmetric information to ensure that general training is optimally provided, given a potential trade off between productive and allocative efficiency?
We have presented a simple model of general training in which the extent of asymmetric information in a competitive labour market is endogenous. We have identified conditions under which an asymmetric information structure arises in equilibrium and (hence) training is efficiently provided. (We say hence here because, under these conditions, training would not be efficiently provided if the available information about the worker’s general human capital was instead symmetric.) Unless the cost of training is too high, the equilibrium outcome achieves the first best: productive efficiency in general training and allocative efficiency in turnover. Thus, for a competitive labour market where firms have full discretion in the design of information structures, our answer to the open question is yes.

There are many ways in which our model could be extended and Section 4 points to several of these. Even with this simple framework, however, it is possible to gain further insight into Pigou’s conjecture, and to shed light on empirical phenomena.
References


